TEXTURE ANALYSIS OF MAMMOGRAMS USING A TWO-DIMENSIONAL AUTOREGRESSIVE MODELLING TECHNIQUE

Sarah Lee and Tania Stathaki

Communications and Signal Processing Group
Department of Electrical and Electronic Engineering
Imperial College London
Exhibition Road, London SW7 2AZ UK

ABSTRACT

The texture of mammograms is characterised using two-dimensional ($2-D$) autoregressive (AR) models under the assumption that each distinct texture can be described by one set of coefficients. In this paper we compare the 1×1 and 2×2 AR model coefficients as well as the degrees of symmetry of these coefficients of the tumour area of a mammogram with other mammograms with the same type of tumour and tissue background. From the simulation results, it is found that the AR model coefficients are similar for the same type of tumours and the degree of symmetry is small for the area containing a tumour, which suggests that the AR model coefficients are symmetric.

1. INTRODUCTION

The two-dimensional ($2-D$) autoregressive (AR) model has been regarded as one of the methods to characterise the textures in an image [2]. A number of methods are available in the literature for solving $2-D$ AR modelling problems including the Yule-Walker system of equations (YW) [3], the Yule-Walker system of equations in the third-order statistical domain (YWT) [7] and the constrained optimisation formulation with equality constraints [4]. In this paper, the last aforementioned method, which relates the YWT method to the YW method is chosen to estimate AR model coefficients, because it is able to give good estimations in both low and high signal-to-noise (SNR) systems and the variances arisen from the estimates of a number of realisations are lower than employing the YWT method alone.

We divide all 322 mammograms in the database [6] into groups according to their tumour type and background tissue characteristics. The details of mammogram are given, such as the character of background tissue, the class and severity of abnormality, the image co-ordinates and the approximate radius of abnormality. The AR model coefficients of the square block containing a tumour are estimated. In [4], we concentrated on the intra-information, i.e., the comparison in AR model coefficients between the tumour block and the blocks in its neighbourhood. Here we look into the inter-information, i.e., how the AR model coefficients are related to different mammograms with the same type of tumour. In this work, we concentrate on the mammograms with fatty background and tumours.

The $2-D$ AR model is defined in Section 2. The YW and YWT methods are revisited in Sections 3 and 4 respectively. In Section 5, the constrained optimisation formulation method used to estimate AR model coefficients is given. The degree of symmetry is defined in Section 6. In Section 7, the simulation results can be found, followed by the conclusion and summary in Section 8.

2. TWO-DIMENSIONAL AUTOREGRESSIVE MODEL

Let us consider a digitised image x of size $M \times N$. Each pixel of x is characterised by its location $[m, n]$ and can be represented as $x[m, n]$, where $1 \leq m \leq M$, $1 \leq n \leq N$. $x[m, n]$ is a positive intensity (gray level). The two-dimensional ($2-D$) autoregressive (AR) model output, $x[m, n]$, is defined as [3]

$$x[m,n] = -\sum_{i=0}^{p_1}\sum_{j=0}^{p_2} a[i,j]x[m-i,n-j] + w[m,n], \quad (1)$$

where $[i, j] \neq [0, 0]$, $a[i, j]$ is the AR model coefficient, $w[m, n]$ is the input driving noise, and $p_1 \times p_2$ is the order of the model.

The driving noise, $w[m, n]$, is non-Gaussian and assumed to be zero-mean, i.e., $E\{w[m, n]\} = 0$, where $E\{\cdot\}$ is the expectation operation. The AR model coefficient $a[0, 0]$ is assumed to be 1 for scaling purpose, therefore we have $[(p_1 + 1)(p_2 + 1) - 1]$ unknown coefficients to solve.

An additional zero-mean Gaussian noise, $v[m, n]$, with variance equal to unity, is added onto the system. Mathematically the new system is written as $y[m, n] = x[m, n]+v[m, n]$. The signal-to-noise ratio (SNR) of the system is calculated by

$$SNR = 10log_{10}\frac{\sigma_x^2}{\sigma_v^2} \qquad dB \quad (2)$$

where σ_x^2 is the variance of the signal and σ_v^2 is the variance of the noise.

3. YULE-WALKER SYSTEM OF EQUATIONS

The Yule-Walker system of equations for the quarter-plane model can be written in matrix form as [3]

$$\underline{\boldsymbol{R}}_{yy}\underline{\boldsymbol{a}}_1 = \underline{\boldsymbol{h}}, \quad (3)$$

where $\underline{\boldsymbol{R}}_{yy}$ is of dimension $[(p_1+1)(p_2+1)] \times [(p_1+1)(p_2+1)]$ and $\underline{\boldsymbol{a}}_1$ and $\underline{\boldsymbol{h}}$ are both of dimension $(p_1 + 1)(p_2 + 1) \times 1$.

More explicitly, equation (3) can be rewritten as in equation (4) on the next page [7]. In equation (4), $\boldsymbol{a}[i] = [a[i, 0], a[i, 1], \cdots, a[i, p_2]]^T$ is a $(p_2 + 1) \times 1$ vector, T denotes transpose, $\boldsymbol{h}_1 =$

$$\begin{pmatrix} \boldsymbol{R}_{yy}[0] & \boldsymbol{R}_{yy}[-1] & \cdots & \boldsymbol{R}_{yy}[-p_1] \\ \boldsymbol{R}_{yy}[1] & \boldsymbol{R}_{yy}[0] & \cdots & \boldsymbol{R}_{yy}[-(p_1-1)] \\ \vdots & \vdots & \ddots & \vdots \\ \boldsymbol{R}_{yy}[p_1] & \boldsymbol{R}_{yy}[p_1-1] & \cdots & \boldsymbol{R}_{yy}[0] \end{pmatrix} \begin{pmatrix} \boldsymbol{a}[0] \\ \boldsymbol{a}[1] \\ \vdots \\ \boldsymbol{a}[p_1] \end{pmatrix} = \begin{pmatrix} \sigma_w^2 \boldsymbol{h}_1 \\ \boldsymbol{0} \\ \vdots \\ \boldsymbol{0} \end{pmatrix} \tag{4}$$

$[1, 0, \cdots, 0]^T$ is a $(p_2 + 1) \times 1$ vector, $\boldsymbol{0} = [0, 0, \cdots, 0]$ is a $(p_2 + 1) \times 1$ vector and
$\boldsymbol{R}_{yy}[i] =$
$$\begin{pmatrix} r_{yy}[i,0] & r_{yy}[i,-1] & \cdots & r_{yy}[i,-p_2] \\ r_{yy}[i,1] & r_{yy}[i,0] & \cdots & r_{yy}[i,-(p_2-1)] \\ \vdots & \vdots & \ddots & \vdots \\ r_{yy}[i,p_2] & r_{xx}[i,p_2-1] & \cdots & r_{yy}[i,0] \end{pmatrix}$$
is a $(p_2 + 1) \times (p_2 + 1)$ matrix.

The second-order moment sample r_{yy} is defined as

$$r_{yy}[i,j] = E\{y[m,n]y[m+i,n+j]\}. \tag{5}$$

Since the input variance σ_w^2 is unknown, the first equation in (4) may be eliminated. The coefficient $a[0,0]$ is assumed to be 1, therefore we can move the first column of the matrix of the remaining system to the right-hand side of the system. Let us write the revised system as

$$\underline{\boldsymbol{R}}\underline{\boldsymbol{a}} = -\underline{\boldsymbol{r}}, \tag{6}$$

where $\underline{\boldsymbol{R}}$ is a $(p_1p_2+p_1+p_2)\times(p_1p_2+p_1+p_2)$ matrix of moments, $\underline{\boldsymbol{a}}$ is a $(p_1p_2 + p_1 + p_2) \times 1$ vector of unknown coefficients and $\underline{\boldsymbol{r}}$ is a $(p_1p_2 + p_1 + p_2) \times 1$ vector of moments.

4. YULE-WALKER SYSTEM OF EQUATIONS IN THE THIRD-ORDER STATISTICAL DOMAIN

The YW method introduced in Section 3 is able to estimate the AR model coefficients only when the system SNR is large. When the SNR is small, the estimation results are influenced by the large Gaussian noise [4, 5], [7] and [8]. In the literature, the Yule-Walker system of equations in the third-order statistical domain has been used to solve this problem. The equations may be written in matrix form as [7]

$$\underline{\boldsymbol{C}}_{yy}\underline{\boldsymbol{a}}_1 = -\underline{\boldsymbol{c}}_{yy}. \tag{7}$$

More explicitly, equation (7) can be rewritten as equation (8) on the next page, in which $\boldsymbol{a}[i] = [a[i,0], a[i,1], \cdots, a[i,p_2]]^T$ is of dimension $(p_2 + 1) \times 1$, $\boldsymbol{h}_1 = [1, 0, \cdots, 0]^T$ is of dimension $(p_2+1)\times 1$, $\boldsymbol{0} = [0, 0, \cdots, 0]^T$ is of dimension $(p_2+1)\times 1$ and the matrix $\underline{\boldsymbol{C}}_{3y}[i]$ of dimensions $(p_2 + 1) \times (p_2 + 1)$ is shown in equation (8).

The third-order moment sample for a zero-mean process may be estimated by
$C_{3y}([i_1, j_1], [i_2, j_2]) =$

$$E\{y[m,n]y[m+i_1,n+j_1]y[m+i_2,n+j_2]\}. \tag{9}$$

The skewness of the driving input is also unknown, therefore we can further simplify the equations applying the same rule mentioned at the end of last section. The equations can be written as

$$\underline{\boldsymbol{C}}\underline{\boldsymbol{a}} = -\underline{\boldsymbol{c}} \tag{10}$$

for model order $p_1 \times p_2$, where $\underline{\boldsymbol{C}}$ is a $(p_1p_2 + p_1 + p_2) \times (p_1p_2 + p_1+p_2)$ matrix of third-order moments, $\underline{\boldsymbol{a}}$ is a $(p_1p_2+p_1+p_2)\times 1$ vector of unknown AR model coefficients and $\underline{\boldsymbol{c}}$ is a $(p_1p_2 + p_1 + p_2) \times 1$ vector of third-order moments.

5. CONSTRAINED OPTIMISATION WITH EQUALITY CONSTRAINTS

The method proposed in [4, 5] relates equation (10) to equation (6) through a constrained optimisation formulation, which can be written mathematically as [4]

$$minimise \sum_{i=1}^{W} (R_i\underline{\boldsymbol{a}} + r_i)^2$$

subject to

$$\underline{\boldsymbol{C}}_1\underline{\boldsymbol{a}} = -\underline{\boldsymbol{c}}_1 \tag{11}$$

where W is the number of rows in matrix $\underline{\boldsymbol{R}}$ in equation (6),
R_i is the i-th row of the matrix $\underline{\boldsymbol{R}}$ in equation (6),
r_i is the i-th element of the vector $\underline{\boldsymbol{r}}$ in equation (6),
$\underline{\boldsymbol{a}}$ is the vector of unknown AR model parameters,
$\underline{\boldsymbol{C}}_1$ is as the matrix $\underline{\boldsymbol{C}}$ defined in equation (10) without the last row, and $\underline{\boldsymbol{c}}_1$ is as the vector $\underline{\boldsymbol{c}}$ derived in equation (10) without the last row.

The last rows in $\underline{\boldsymbol{C}}$ and $\underline{\boldsymbol{c}}$ are chosen to be eliminated after some statistical tests. Equation (11) is solved using sequential quadratic programming (SQP) technique [1].

6. DEGREE OF SYMMETRY

The degree of symmetry (DOS) was first defined in [4] for 1×1 AR models under the assumption that a set of stable $2 - D$ AR model of coefficients is formed by two sets of stable $1 - D$ AR models. If $\underline{a}_1$ is a row vector that represents a set of a stable $1-D$ AR model coefficients and $\underline{a}_2$ is another row vector that represents a set of stable $1 - D$ AR model coefficients, then $\underline{a}$, where $\underline{a} = \underline{a}_1^T \times \underline{a}_2$, is a set of stable $2 - D$ AR model coefficients and T denotes the transposition. When $\underline{a}_1$ is equal to $\underline{a}_2$, the $2 - D$ AR model coefficients representing by the matrix $\underline{a}$ is symmetric. Mathematically the DOS is written as

$$\mathrm{DOS}_{1\times 1} = a[1,1] - a[0,1] \times a[1,0]. \tag{12}$$

In this paper, the idea is extended for 2×2 AR models.
$\mathrm{DOS}_{2\times 2} =$

$$\frac{1}{4}\left[(a[1,1] - a[0,1] \times a[1,0] + (a[1,2] - a[0,2] \times a[1,0]) + (a[2,1] - a[0,1] \times a[2,0]) + (a[2,2] - a[0,2] \times a[2,0])\right] \tag{13}$$

When the DOS is small for an AR model, it is said to be more symmetric. For an absolute symmetric AR model, the DOS is zero.

7. RESULTS FROM MAMMOGRAMS

322 mammograms can be obtained from the MIAS MiniMammographic Database [6], from which we are also given the details of the mammograms such as the character of background tissue: fatty, fatty-glandular or dense-glandualr; class of abnormality present: calcification, well-defined or circumscribed masses,

$$
\begin{pmatrix} \underline{\boldsymbol{C}}_{3y}[0] & \underline{\boldsymbol{C}}_{3y}[-1] & \cdots & \underline{\boldsymbol{C}}_{3y}[-p_1] \\ \underline{\boldsymbol{C}}_{3y}[1] & \underline{\boldsymbol{C}}_{3y}[0] & \cdots & \underline{\boldsymbol{C}}_{3y}[-(p_1-1)] \\ \vdots & \vdots & \ddots & \vdots \\ \underline{\boldsymbol{C}}_{3y}[p_1] & \underline{\boldsymbol{C}}_{3y}[p_1-1] & \cdots & \underline{\boldsymbol{C}}_{3y}[0] \end{pmatrix} \begin{pmatrix} \boldsymbol{a}[0] \\ \boldsymbol{a}[1] \\ \vdots \\ \boldsymbol{a}[p_1] \end{pmatrix} = \gamma_w \begin{pmatrix} \boldsymbol{h}_1 \\ \boldsymbol{0} \\ \vdots \\ \boldsymbol{0} \end{pmatrix} \tag{8}
$$

$$
\underline{\boldsymbol{C}}_{3y}[i] = \begin{pmatrix} C_{3y}([i,0],[i,0]) & C_{3y}([i,-1],[i,-1]) & \cdots & C_{3y}([i,-p_2],[i,-p_2]) \\ C_{3y}([i,1],[i,1]) & C_{3y}([i,0],[i,0]) & \cdots & C_{3y}([i,-(p_2-1)],[i,-(p_2-1)]) \\ \vdots & \vdots & \ddots & \vdots \\ C_{3y}([i,p_2],[i,p_2]) & C_{3y}([i,p_2-1],[i,p_2-1]) & \cdots & C_{3y}([i,0],[i,0]) \end{pmatrix}
$$

spiculated masses, architectural distoration, asymmetry, normal or other ill-defined masses; severity of abnormality: benign or malignant; image co-ordinates of centre of abnormality; and approximate radius of abnormality. The mammograms remain labelled using the index numbers in the database.

We divide all mammograms into groups according to the background tissue, the class of abnormality and the severity of the mass. For each group, a set of 1×1 and a set of 2×2 AR model coefficients are estimated for the area containing the tumour using the method mentioned in Section 5. We present the results of two groups, which are mammograms with a well-defined benign tumour and fatty background tissue (Group F CIRC B) and mammograms with an ill-defined malignant mass and fatty background tissue (Group F MISC M). There are 9 mammograms in Group F CIRC B with 2 of them having two tumours in the same mammogram and there are 5 mammograms in Group F MISC M.

The estimated 1×1 AR model coefficients for each tumour in Group F CIRC B may be found in Figure 1. The estimated 2×2 AR model coefficients for this group are shown in Figure 2. The results for Group F MISC M are displayed in Figures 3 and 4 for 1×1 and 2×2 AR model coefficients respectively. The degree of symmetry of both 1×1 and 2×2 AR models for Groups F CIRC B and F MISC M may be found in Tables 1 and 2 respectively.

7.1. Analysis of Results

In Figure 1, all mammograms in the database with F CIRC B characteristics have similar 1×1 AR model coefficient values except mdb010. The similar pattern can be found in Figure 2 for 2×2 AR model coefficients estimated. Mammogram mdb010 has smaller coefficient values again. However, the coefficient $a[2,2]$ fails to provide the same pattern. In Figures 3 and 4, the mammograms with a malignant tumour and fatty background tissue demonstrated their similarity in 1×1 AR model coefficients. For the 2×2 AR model coefficients, mdb274 have larger values in coefficients $a[1,2]$ and $a[2,1]$.

As shown in Table 1, all mammograms in Group F CIRC B except mdb010 fall in the range of $-0.0098 \leq \mathrm{DOS}_{1\times 1} \leq -0.0015$. The AR model coefficients representing the tumours are more symmetric in AR model order 2×2 than order 1×1, since $\mathrm{DOS}_{2\times 2}$ are much smaller than $\mathrm{DOS}_{1\times 1}$. This also applies to the Group F MISC M as shown in Table 2.

8. CONCLUSION AND SUMMARY

In this paper, we revisited three conventional methods for two-dimensional autoregressive model coefficient estimation: the Yule-Walker system of equations, the Yule-Walker system of equations and the constrained optimisation formulation with equality constraints. We applied the last method to compute the AR model coefficients of tumours in mammograms with fatty-background. From the simulation results obtained, it can be concluded that the same type of tumours have similar AR model coefficients in both order 1×1 and 2×2 and the AR model coefficients are more symmetric in order 2×2 than order 1×1.

9. REFERENCES

[1] P.E. Gill, W. Murray, and M.H. Wright, *Practical Optimization*, Academic Press, 1981.

[2] R.M. Haralick, “Statistical and Structural Approaches to Texture”, *Proc. IEEE*, Vol. 67, No. 5, pp. 786-804, May 1979.

[3] S.M. Kay, Modern Spectral Estimation: Theory and Application, Prentice Hall 1988.

[4] S. Lee and T. Stathaki, “Texture Characterisation Using Constrained Optimisation Techniques with Application to Mammography”, *Fifth International Workshop on Image Analysis for Multimedia Interactive Services (WIAMIS 2004)*, on CD-ROM, April 2004.

[5] S. Lee and T. Stathaki, chapter titled, “Characterization of texture surfaces in medical imaging”, *Applied Medical Image Analysis Methods*, Editor: Lena Costaridou, CRC Press, to be published, 2005.

[6] The MIAS MiniMammographic Database, ftp://peipa.essex.ac.uk/ipa/pix/mias/, last access on 1st February 2005.

[7] P.T. Stathaki, *Cumulant-Based and Algebraic Techniques for Signal Modelling*, Ph. D. Thesis, Imperial College London, 1994.

[8] A. Swami, G.B. Giannakis and J.M. Mendel, “Linear Modeling of Multidimensional Non-Gaussian Pro- cesses Using Cumulants”, *Multidimensional Systems and Signal Processing*, Vol. 1, pp. 11-37, 1990.

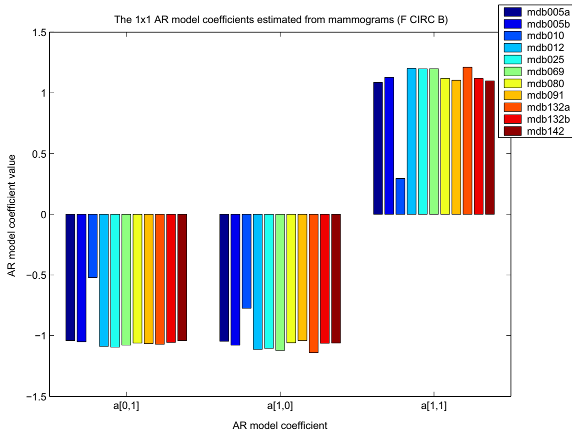

Fig. 1. The 1×1 AR model coefficients estimated from mammograms in which there is a circumscribed benign tumour with fatty background tissue (Group F CIRC B).

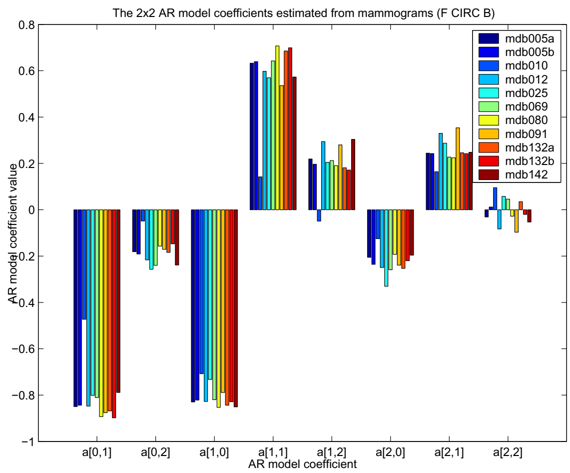

Fig. 2. The 2×2 AR model coefficients estimated from mammograms in which there is a circumscribed benign tumour with fatty background tissue (Group F CIRC B).

Mammogram	$\mathbf{DOS}_{1\times1}$	$\mathbf{DOS}_{2\times2}$
Mdb005a	-0.0015	-1.6383×10^{-4}
Mdb005b	-0.0036	-4.3223×10^{-4}
Mdb010	-0.1081	-0.0200
Mdb012	-0.0098	-0.0012
Mdb025	-0.0096	-8.8001×10^{-4}
Mdb069	-0.0092	-9.6143×10^{-4}
Mdb080	-0.0029	-4.3219×10^{-4}
Mdb091	-0.0023	-2.9516×10^{-4}
Mdb132a	-0.0095	-0.0012
Mdb132b	-0.0031	-4.3966×10^{-4}
Mdb142	-0.0023	-2.8753×10^{-4}

Table 1. Degree of symmetry (DOS) of estimated 1×1 and 2×2 AR model coefficients from Group F CIRC B.

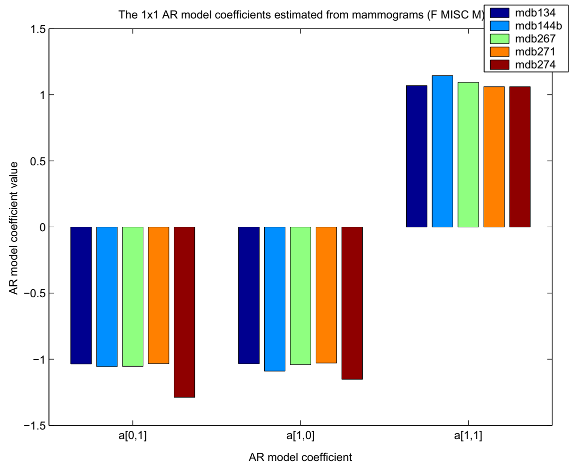

Fig. 3. The 1×1 AR model coefficients estimated from mammograms in which there is a malignant tumour with fatty background tissue (Group F MISC M).

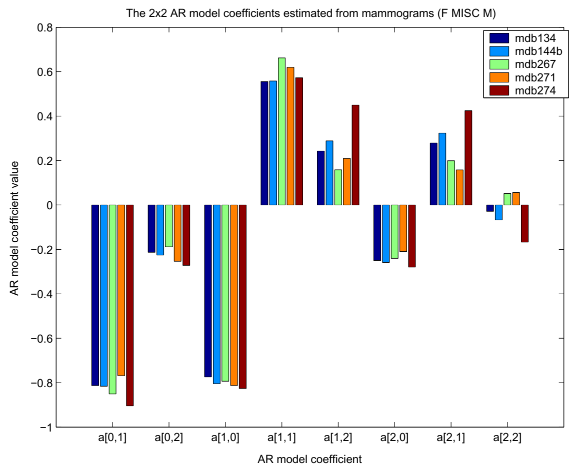

Fig. 4. The 2×2 AR model coefficients estimated from mammograms in which there is a malignant tumour with fatty background tissue (Group F MISC M).

Mammogram	$\mathbf{DOS}_{1\times1}$	$\mathbf{DOS}_{2\times2}$
Mdb134	-0.0011	-1.5109×10^{-4}
Mdb144b	-0.0050	-6.1454×10^{-4}
Mdb267	-0.0019	-3.1942×10^{-4}
Mdb271	-7.2625×10^{-4}	-9.3807×10^{-4}
Mdb274	-0.0438	-0.0046

Table 2. Degree of symmetry (DOS) of estimated 1×1 and 2×2 AR model coefficients from Group F MISC M.