CONTOUR METRIC BASED IN THE CURVATURE EXTREMES OF THE DIFFUSED CONTOUR

António M. G. Pinheiro

Unidade de Detecção Remota
Universidade da Beira Interior
6200 Covilhã - Portugal
E-mail: pinheiro@dfisica.ubi.pt

ABSTRACT

The distance of the relative positions of the curvature extremes of two shape contours are used as the similarity measure between two shapes. The shape contours are first smoothed through Gaussian filtering. So, the shape contours are represented at a given scale, where small geometric features have been suppressed. The curvature extremes will define a set of descriptors for the shape contours. As in the human visual system they are suitable for contour comparison. A reliable shape based retrieval technique is then defined. This shape comparison method is also compared with others. Some advantages are identified. Results that show the reliability of this method are shown.

Keywords: Shape, Contour, Similarity Metric, Shape Retrieval, Scale, Curvature.

1. INTRODUCTION

Shape comparison techniques are used in pattern recognition and in image retrieval. Together, with color and texture analysis provide very reliable tools for image recognition.

Humans tend to make an image comparison based in their objects. This is because objects are one of the first things detected by the human inspection of images. So, the similarity between two images tends to be defined in function of the similarity of their objects. It is important to define methods that allow shape comparison, because shape is a very important object descriptor.

A shape comparison method must match our intuitive notion of shape resemblance. It should be invariant to translation, rotation and uniform scaling. There are many techniques for shape comparison purposes. Some shape comparison methods compute the similarity between two shapes based on the shape geometry similarity [1]. However, if one of the object suffers some kind of deformation, these kind of methods become usually useless. The solution is to detect feature points, that some how, define the main visual properties of the shape. A very common solution results of comparing the maxima of the contours' curvature zero crossing of the scale space image [2]. These maxima will then define a contour descriptor suitable for comparison. Among the several approaches for shape description and comparison must also be distinguished the Fourier descriptors [3].

In our work, we propose to replace the curvature zero crossing of the scale space image maxima [2], by the extremes of the curvature after gaussian smoothing of the contour. Using a proper strategy, a reliable and efficient shape metric is defined. This method is applied in a shape based retrieving scenario. Very promising results were obtained using the SQUID data-base [4] with 1100 contours.

2. SCALE SPACE REPRESENTATION OF A SHAPE CONTOUR

Consider the shape contour as a close plane trajectory ℓ of a point. If x and y are the plane coordinates, the path ℓ can be represented by [5, 6]:

$$\ell : S \subseteq \Re \rightarrow \Re^2,\ \ell(s) = [x(s), y(s)] \tag{1}$$

where the parameter s is chosen to be the arclength of the curve. A scale space representation of the contour can be obtained by the diffusion equation:

$$\ell_t = \nabla.(\tau \nabla \ell), \tag{2}$$

where the original contour $\ell(s, 0)$ is used as an initial condition. In the simplest form τ is a constant, resulting the linear diffusion equation

$$\ell_t = \tau \Delta \ell, \tag{3}$$

that leads to the following solution at each scale t:

$$\ell(s,t) = \begin{cases} \ell(s,0) & \text{if} \quad t = 0 \\ (G_{\sqrt{2t}}(s) * \ell(s,0)) & \text{if} \quad t > 0 \end{cases}, \tag{4}$$

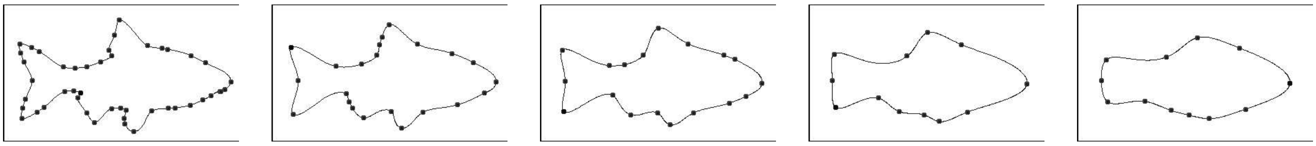

Fig. 1. Extremes for the scales t = 32, 128, 256, 512 e 1024.

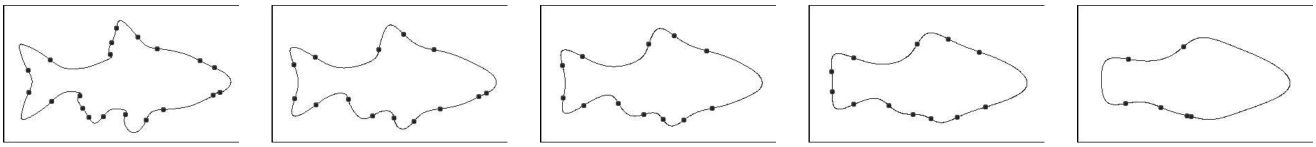

Fig. 2. Zero crossings for the scales t = 32, 128, 256, 512 e 1024.

where $G_\sigma = (1/2\pi\sigma^2)\exp(-s^2/2\sigma^2)$ is the Gaussian function with a standard deviation $\sigma = \sqrt{2t}$ and $*$ represents the convolution operation. Equation 4 can be expressed in terms of two components:

$$\ell(s,t) = [X(s,t), Y(s,t)], \tag{5}$$

where $[X(s,t), Y(s,t)] = [G_{\sqrt{2t}} * x(s), G_{\sqrt{2t}} * y(s)]$.

The resulting trajectories $\ell(s,t)$ represent smooth versions of the original contour in a scale t. Higher scales (or lower resolutions) result in smoother versions of the original contour.

The curvature function $\kappa_\ell(s,t)$ characterizes the change of the direction of a curve $\ell(s,t)$. At a given scale t, the curvature function is defined as the instantaneous rate of change of the tangent $\theta(s,t)$ of $\ell(s,t)$ at point s:

$$\kappa_\ell(s,t) = \frac{d\theta(s,t)}{ds} \tag{6}$$

The main feature points can be detected at any scale by using (6). By finding the local curvature extremes, we are able to identify the contour positions where sharp changes in the contour direction occur. The curvature zero crossings define an inflection of the contour direction (change in the contour concavity).

These points of the curvature define important geometric characteristics of the contour, that have important properties expected from the scale space theory developed in [7, 8]: 1) Extremes or zero crossings in higher scales (lower resolutions) identify more important geometric properties. Due to the stronger smoothness, these points have global geometric importance. When these feature points are only detected in lower scales only represent local geometric characteristics of the contour. 2) When they are present in higher scales (low resolutions) they always exist in the lower scales (high resolutions). Another important property is related to the fact that along the scale, extremes and zero crossings have always similar positions, and can easily be tracked along the scale.

3. THE SHAPE SIMILARITY METRIC

The main motivation of this work results from the fact that maxima of the curvature represent very important feature points for the human visual comparison of shapes. Extremes correspond to the salient contour points (see figure 1). The human visual system tends to be more sensitive to this feature, in opposition to the position of the zero crossings of the curvature. As they represent points where the concavity of the contour changes (see figure 2), they are ideal points for contour segmentation [9].

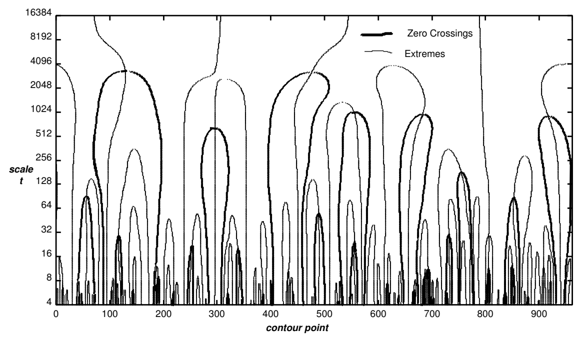

Fig. 3. Scale space image of the zero crossings and of the extremes for the contour of figure 1.

But, is there a huge difference between the curvature extremes position and the maxima of the curvature zero crossings scale space image? In fact, there is not, as it is shown in the graphic of figure 3. The maxima of the curvature zero crossing scale space maps are in similar positions to the curvature extremes. Moreover, extremes have other kinds of information associated, like if they are maxima or minima, or what is the curvature value.

In this method, the curvature of the smoothed contour in a scale t are computed. Then, are defined a set of points that represent the curvature value in each position where a curvature extreme exists. These points can be represented by a set of points represented by $(\hat{\kappa_\ell}, \hat{s})$ is defined. There $\hat{\kappa_\ell}$ represents the curvature normalized to one, and $\hat{s}$ the arclength of the curve normalized to one too. With these normalizations the method becomes invariant to the uniform scaling of the contour.

The above formulation leads to a shape metric given by the summation of the euclidean distances between the points where the curvature extremes are located, $(\hat{\kappa_\ell}, \hat{s})$, of the two contours under comparison:

$$\mathcal{S} = \sum d\left((\hat{\kappa_{\ell_1}}, \hat{s_1}), (\hat{\kappa_{\ell_2}}, \hat{s_2})\right) \tag{7}$$

where 1 and 2 represent each of the contours.

The distance between the extremes of the two contours $(\hat{\kappa_\ell}, \hat{s})$ is defined in a neighbourhood $\Delta\hat{s}$. The two curvature extremes must be of the same kind (maxima or minima) and have values within a $\Delta\hat{\kappa_\ell}$. In this way, only the same kind of extremes are compared. When a curvature extreme of one of the contours does not have a correspondent one, a weighted euclidean distance of $(\hat{\kappa_\ell}, \Delta\hat{s})$ with (0,0) is computed. This missing case is only very serious if the curvature has a relatively high absolute value. Otherwise, only represents a small detail of the contour, and can't weight too much in the final $\mathcal{S}$ value.

For matching purposes, all the combinations of starting points of both contours should be compared. However, such a procedure becomes of very heavy computation. In retrieving environments, where each contour is compared with a whole data-base of contours, the efficiency degradation would become very serious. To avoid this situation, only the combinations where the main curvature extremes match between them, are considered. The experimental tests, leads to the conclusion that this limitation will improve in most of the cases the quality of the matching results, while improves the computational efficiency by hundreds of times.

4. RESULTS

As can be seen in figure 1 and in the graphic of figure 3, a scale t increase, results in less extremes. So, higher scales will lead to a less detailed comparison, while lower scales leads to more detailed comparison. However, low scales result in several extremes that represent local geometric features. These geometric features are in general not important for contour comparison purposes. So, it is chosen a relatively high scale, that keeps the most important geometric details of the contour. A good example of such a scale is t=512 (see figure 1).

The smoothing scale is also a factor that can influence the shape comparison when there is a uniform scaling. To avoid different degrees of smoothing, the scale t is computed proporcionally to the dimension of the contour. So, a scale t will be considered from now on, as the smoothing scale of a contour with a 1000 points. The real smoothing scale will be given by $t \times 1000/N$, where N is the number of points of the contour.

With these considerations, several experiences using the described method have been made. In general, good results are obtained.

In figure 4 are shown examples of shape based retrieving, using the SQUID data-base [4]. From a subjective point of view, the results are very good. They are representative of several experiences that have been achieved. From the different results it is concluded that the defined technique performs very well when used in shape based retrieving environments.

The results show the independence of the method with the rotation and uniform scaling. The last row show an example that reveals the robustness against deformations of the initial shape.

This method is also of very efficient computation. The search for the first point of the contours is very fast because are only used a small set of possibilities, that result in a match of the extremes with higher absolute value.

The descriptors computation are also of efficient computation, because only a filtering step is necessary. For the curvature zero crossing maxima of the scale space image, filtering must be done in several frequencies, until an acceptable scale space image is build. The resulting image must allow the correct extraction of the maxima positions.

The results shown were obtained with fixed parameters. Were used the following values: scale t = 512, neighbourhood $\Delta\hat{s}$ = 0.1 and maximum difference between compared extremes $\Delta\hat{\kappa_\ell}$ = 30%.

Change of the parameters lead to different results. Those change can be explained by the parameter meaning. So, a tiny window $\Delta\hat{s}$ results in a more selective retrieving, relatively to the difference of the elongation of the shape contours between salient points. Higher values of $\Delta\hat{\kappa_\ell}$ makes the retrieving process more selective relatively to the saliency of each change of the contour direction.

Also interesting is to watch the difference of the curvature graphics. As an example, the last match of the middle row (figure 5). The query contour (left one) is matched with the fifth retrieved (right one). The graphics represent the situation that leads to the matching position.

5. SUMMARY

The described technique has revealed to be a very robust, reliable and efficient shape comparison, suitable for shape based retrieval.

By using the extremes of the contour curvature, this technique uses a descriptor that defines important feature points. The human visual system is very sensitive to this

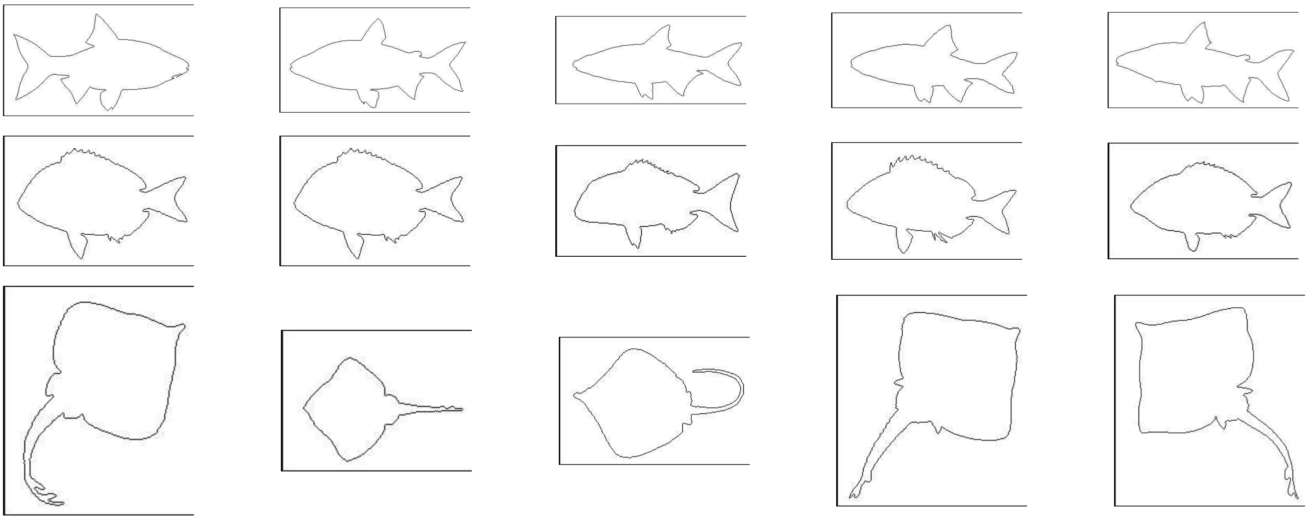

Fig. 4. Examples of shape based retrieving. From the left column to the right: query contour and first retrieved followed by the most similar contours, in a similarity decreasing order.

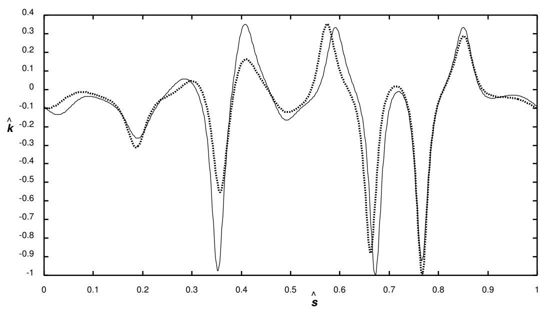

Fig. 5. Curvature graphics in the matching position of the contours on the left and on the right of the middle row of 4.

geometric feature. Because of that, the retrieved results appear so close to the human notion of similarity.

This paper defines a technique for shape comparison that is very competitive with the most well known ones.

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